

# A New Approach to Chiral Perturbation Theory with Matter Fields

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(July 9, 1996)

## Abstract

I propose a prescription for separating the high- and low-energy contributions in effective field theories. This prescription allows a relativistic treatment of matter fields in chiral perturbation theory while the power counting remains valid.

PACS number(s): 12.39.Fe, 11.10.Gh

Quantum chromodynamics (QCD) is the underlying theory of low-energy effective field theories (EFTs). Despite the presence of an infinite number of free parameters, an EFT may provide useful perturbative calculations in powers of the energy [1,2]. Underlying the low-energy expansion is the power counting of Weinberg [1,3].

In general, EFTs permit useful low-energy expansions only if we absorb *all* the hard-momentum effects into the parameters of the lagrangian. For chiral perturbation theory (CHPT) in the pion sector [2], the only hard-momentum effects are the ultraviolet divergences, which are absorbed into the parameters through renormalization. When we include the nucleons relativistically, the anti-nucleon contributions are also hard-momentum effects. These hard effects were not absorbed into the parameters of the lagrangian in the relativistic treatment of Gasser, Sainio, and Svarc (GSS) [4]. As a result, the power counting fails and a systematic expansion in energy is not possible. In the heavy-baryon formalism, Jenkins and Manohar [5] recover the power counting by integrating out the anti-nucleon field so that its effect is absorbed into the parameters. Weinberg [3] has also introduced a similar nonrelativistic formalism for the nucleons in discussing the nucleon–nucleon potential.

Although heavy-baryon ChPT has been successful in many applications, it is useful to have a relativistic formalism in which the power counting is still valid [6]. First, we do not need to keep track of various sets of terms of  $1/M$  corrections resulting from the nonrelativistic reduction of the lagrangian [7], where  $M$  is the nucleon mass. Next, a relativistic formalism may provide new insights into the momentum expansion. For example, as demonstrated later, I find that loops may generate divergences to all orders without jeopardizing the power counting—a claim that may surprise some practitioners in CHPT. Finally, a relativistic description also significantly simplifies the finite-density problem, where the  $1/M$  corrections as well as the differences between the scalar density  $\langle \bar{N}N \rangle$  and the vector density  $\langle N^\dagger N \rangle$  are important. Indeed, the relativistic many-body hadronic theory, so-called quantum hadrodynamics (QHD) [8], has been quite efficient for calculations in nuclear matter and finite nuclei.

To achieve a useful relativistic formalism with matter fields (i.e. those of heavy hadrons),

we must find some unambiguous prescription to separate the hard- and soft-momentum effects so that we can absorb the former into the parameters while evaluating the latter. We may consider the absorption of the hard-momentum parts of any Feynman diagrams into the parameters as an extra renormalization. This extra renormalization is necessary for a useful low-energy expansion because we have a theory that contains not only Goldstone bosons whose masses are much lighter than the large-energy scale, but also baryons whose masses are of the same order as this scale. Note that the idea of absorbing vacuum effects of heavy hadrons into the parameters has been adopted in Ref. [9], where the vacuum one-baryon-loop energy of nuclear matter and finite nuclei are absorbed into the scalar potential. In this Letter, I propose a prescription for separating the hard- and soft-momentum contributions in any Feynman diagrams so that we can perform the extra renormalization. I show that the soft part of any Feynman diagram satisfies Weinberg's power counting, so a systematic low-energy expansion becomes possible even in the relativistic theory with matter fields.

We start with the same relativistic chiral lagrangian as in GSS and use the equations of motion or field redefinitions to remove redundant terms [3,10,11]. The lagrangian without external sources and with the inclusion of the  $\Delta$  isobar can be found in Refs. [14,15]. Thus, we derive the Feynman rules just as usual. We do not deviate from GSS until we deal with the loop integrals and perform the extra renormalization which absorbs the hard parts into the parameters of the lagrangian. GSS keep the large hard parts that spoil the power counting. Jenkins and Manohar [5] modify the relativistic lagrangian by integrating out the anti-nucleon field. In this heavy-baryon formalism, they achieve a useful low-energy expansion. However, their lagrangian is much more involved than the original relativistic one [7]. Our approach will produce the same results as the heavy-baryon formalism if the  $1/M$  corrections are included appropriately in the latter. The advantage of our approach is that it enjoys the simplicity of the relativistic lagrangian as in the treatment of GSS, while it also allows a consistent low-energy expansion as in the heavy-baryon formalism.

I emphasize that loops may generate divergences to all orders so that we may require renormalization to all orders at any loop order. This requirement does not cause problems

because the truncation of the lagrangian must be based on renormalized coefficients; it is these renormalized coefficients that may satisfy the naive dimensional analysis [12,13]. We simply need to retain contact terms up to the same order as that of the renormalized loop contributions, rather than up to the highest order in which the loops generate divergences.

Let  $Q$  stand for a generic soft-momentum scale of the order of the pion mass  $m_\pi$  and take the nucleon mass  $M$  to represent the typical large-energy scale. We are interested in applications where the space components of external nucleons are of order  $Q$  and so is the mass splitting  $(M_\Delta - M)$  between the nucleon and the  $\Delta$ -isobar. Quantities of order  $M$  include the  $\Delta$  mass and the factor  $4\pi f_\pi$  with  $4\pi$  coming from a loop integral [12] and  $f_\pi \approx 93 \text{ MeV}$  being the pion decay constant.

We can write the integral  $G$  for a Feynman graph as the sum of its hard and soft parts:

$$G = (G - \hat{R}\hat{S}G) + \hat{R}\hat{S}G , \quad (1)$$

where  $\hat{S}$  is an operator that extracts the unrenormalized “soft” part of  $G$  and  $\hat{R}$  is an operator that renormalizes  $\hat{S}G$  in the standard way so as to remove the residual hard contributions (ultraviolet divergences in the form of poles at  $d = 4$  in dimensional regularization). The prescription for obtaining the unrenormalized soft part  $\hat{S}G$  from  $G$  consists of the following rules:

1. Take the loop momenta to be of order  $Q$ .
2. Make a covariant  $Q/M$  expansion of the integrand.
3. Exchange the order of the integration and summation of the resulting power series.

That the prescription indeed extracts the soft part of a Feynman diagram is shown as follows. Let  $q$  be a loop momentum and consider the integration over the time component  $q_0$ . Closing the contour by a semicircle at infinity, we find that the  $q_0$ -integration of  $G$  is given by the sum of three contributions: (1) the semicircle, (2) the soft poles, and (3) the hard poles. Here the soft and hard poles are those at momenta of orders  $Q$  and  $M$  respectively.

We have assumed that we can divide the poles into the soft and hard ones, as is the case for theories with Goldstone bosons and massive baryons. First, we note that the semicircle could produce divergences which will later be removed by the usual renormalization. Next, we note that our prescription does not allow a  $Q/M$  expansion of the soft-pole structures such as the pion propagator. However, we can make a  $Q/M$  expansion of hard-pole structures because the momenta at which the poles locate are much larger than the loop momentum when the latter is taken to be of order  $Q$  by Rule 1. We require a covariant expansion to maintain Lorentz invariance. Finally, integration term by term by Rule 3 removes the contributions of the hard poles since they do not appear in any individual terms. Thus, we are left with the unrenormalized soft part, in which ultraviolet divergences can still occur.

Because the prescription retains all the soft poles, the remainder  $(G - \hat{R}\hat{S}G)$  must have contributions only from large momenta including some ultraviolet divergences. We can write this hard part as local counterterm contributions—a well-known result that is fundamental to the idea of effective field theories. This result is intimately related to the uncertainty principle as argued by Lepage [16]. Indeed, large momenta correspond to short distances that are tiny compared with the wavelengths of the external particles, so the interactions must be local. Thus, we can perform the extra renormalization of absorbing the hard part into the parameters of the lagrangian. That Weinberg’s power counting remains valid in our approach can be shown as follows:

First, we prove that diagrams with closed fermion loops contain no soft parts. Two kinds of fermion loops may occur. The first kind does not contain any nearly on-shell baryon lines. An example is a baryon loop that is not connected to any external baryon lines. In this case, Rules 1 and 2 allow us to expand the fermion propagators as polynomials in the loop momentum. For example, we can expand the nucleon propagator  $G(q)$  as follows:

$$G(q) = \frac{1}{\not{q} - M + i\epsilon} = -\frac{1}{M} \left( 1 + \frac{\not{q}}{M} + \frac{q^2}{M^2} + \cdots \right), \quad (2)$$

where  $q$  is taken to be of order  $Q$  by Rule 1. Thus, integrating term by term using dimensional regularization, we obtain vanishing soft parts. The second kind may involve multiple fermion

loops connected to external and containing internal baryon lines that are nearly on shell. This kind has at least one fermion loop that contains one and only one nearly on-shell baryon line. We note that an internal nearly on-shell baryon propagator carries a momentum  $k + q$  with both  $q$  and  $(k^2 - M^2)/M$  being of order  $Q$ . Thus, for a nucleon like this, we obtain the covariant expansion of its propagator as

$$G(k + q) = \frac{\not{k} + \not{q} + M}{2k \cdot q + k^2 - M^2 + i\epsilon} - \frac{q^2(\not{k} + \not{q} + M)}{(2k \cdot q + k^2 - M^2 + i\epsilon)^2} + \cdots, \quad (3)$$

where the leading term is of order  $1/Q$  and each succeeding term is suppressed by  $Q/M$ . Now for the loop with just one nearly on-shell baryon line, by Rules 1 and 2, we can expand the nearly on-shell propagator as in Eq. (3) and all other baryon propagators as in Eq. (2). Rule 3 then allows us to integrate term by term. The resulting loop integral again vanishes in dimensional regularization since (see Ref. [17] for example)

$$I(k) \equiv \int d^d q \frac{(q^2)^m}{(2k \cdot q + k^2 - M^2 + i\epsilon)^n} = 0 \quad (4)$$

for any integers  $m$  and  $n$  and any derivatives of  $I(k)$  also vanish. Thus, we can ignore fermion loops.

Next, we note that, once the baryon loops are excluded, any baryon lines must be connected to some external baryons by baryon current conservation. Thus, the baryon propagators are nearly on shell and are of order  $1/Q$  from the expansion in Eq. (3). In practice, we truncate Eq. (3) at some order depending on the accuracy to which we calculate the relevant physical quantity. The rest of the power-counting argument becomes the same as in Weinberg's work [3]. It follows that a Feynman diagram with  $L$  loops,  $E_N$  external baryon lines carries the order  $Q^\nu$  with

$$\nu = 2 + 2L - \frac{1}{2}E_N + \sum_i V_i \left( d_i + \frac{1}{2}n_i - 2 \right), \quad (5)$$

where  $V_i$  is the number of vertices of type  $i$  characterized by  $n_i$  baryon fields and  $d_i$  pion derivatives or  $m_\pi$  factors.

We have shown that the prescription can be carried out for any Feynman diagrams. Thus, it can also be carried out for any Green's functions because they are just the sum

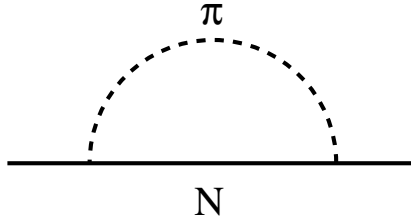


FIG. 1. One-loop self-energy of the nucleon.

of Feynman diagrams constructed from the usual Feynman rules, which are still applicable in our approach. Note that the prescription has no effects on ChPT in the pion sector because there are no hard poles there. Also, in many practical calculations, the hard part is seldom needed because it is cancelled by counterterm contributions. Instead, we evaluate and renormalize the soft part and combine it with the tree contributions, which we truncate up to the same order as the loop contributions.

In what follows, to illustrate how the prescription works, I calculate in detail the nucleon one-loop self-energy with a  $\pi N$  intermediate state, as shown in Fig. 1. Both the hard and soft parts will be evaluated explicitly. The corresponding result in heavy-baryon ChPT can be reproduced in the limit of infinite nucleon mass. I shall also briefly discuss the structure of the  $\pi N$  scattering amplitude up to order  $Q^3$ .

From the Feynman rules derived as usual, the self-energy of a nearly on-shell nucleon of momentum  $k$  is

$$\Sigma_{\pi N}(k) = -\frac{3g_A^2}{4f_\pi^2} i\mu^{4-d} \int \frac{d^d q}{(2\pi)^d} \frac{\not{q}\gamma_5 G(k+q) \not{q}\gamma_5}{q^2 - m_\pi^2 + i\epsilon}, \quad (6)$$

where  $g_A \approx 1.26$  is the axial coupling and  $\mu$  is the scale of dimensional regularization. According to the prescription, we obtain the soft part of  $\Sigma_{\pi N}$  by first making a covariant  $Q/M$  expansion of the integrand while taking the loop momentum  $q$  to be soft. Since we assume  $k$  to be nearly on shell, the expansion of the nucleon propagator is as given in Eq.(3). Exchanging the order of the summation and integration then gives the soft part.

Although we may truncate the expansion in Eq. (3) at some order, we notice in the present case that, after our exchange of the integration and summation and with the use of Eq. (4), the  $q^2$  in the numerator of any integrand can be replaced with  $m_\pi^2$  in dimensional

regularization. After this replacement we may resum the series and postpone the discussion of the  $1/M$  expansion until the integral is performed. Thus, we obtain the one-loop self-energy to all orders in the  $1/M$  expansion:

$$\hat{S}\Sigma_{\pi N}(k) = \frac{3}{4} \frac{g_A^2}{f_\pi^2} i\mu^{4-d} \int \frac{d^d q}{(2\pi)^d} \frac{(\not{k} + M)m_\pi^2 - (2k \cdot q + m_\pi^2)\not{q}}{(q^2 - m_\pi^2 + i\epsilon)(2k \cdot q + k^2 - M^2 + m_\pi^2 + i\epsilon)} . \quad (7)$$

Clearly, the soft part  $\hat{S}\Sigma_{\pi N}$  is of order  $Q^3/M^2$  in agreement with Eq. (5). It is straightforward to perform the integral of Eq. (7) to yield

$$\begin{aligned} \hat{S}\Sigma_{\pi N}(k) = \Sigma_{\pi N}^{\overline{\text{MS}}}(k) + \frac{3g_A^2}{4f_\pi^2} & \left[ \frac{k^2 - M^2}{k^2} (m_\pi^2 - 2\omega^2) \not{k} \right. \\ & \left. + \frac{2\omega m_\pi^2}{\sqrt{k^2}} (\not{k} + M) \right] \left( L + \frac{1}{32\pi^2} \ln \frac{m_\pi^2}{\mu^2} \right) , \end{aligned} \quad (8)$$

$$\begin{aligned} \Sigma_{\pi N}^{\overline{\text{MS}}}(k) = \frac{3g_A^2}{64\pi^2 f_\pi^2} & \left[ \frac{k^2 - M^2}{k^2} \omega \not{k} - \frac{m_\pi^2}{\sqrt{k^2}} (\not{k} + M) \right] \\ & \times \left( \omega + 2\sqrt{m_\pi^2 - \omega^2} \tan^{-1} \frac{\sqrt{m_\pi^2 - \omega^2}}{\omega} \right) , \end{aligned} \quad (9)$$

where we introduce the following notation:

$$\omega \equiv \frac{1}{2\sqrt{k^2}} (k^2 - M^2 + m_\pi^2) , \quad (10)$$

$$L \equiv \frac{1}{32\pi^2} \left( \frac{2}{d-4} + \gamma - 1 - \ln 4\pi \right) , \quad (11)$$

with Euler's constant  $\gamma = 0.577 \dots$ . As seen from Eq. (8),  $\Sigma_{\pi N}^{\overline{\text{MS}}}$  is the self-energy renormalized at scale  $\mu = m_\pi$  in the modified minimal subtraction ( $\overline{\text{MS}}$ ) scheme.

Expanding the second term of Eq. (8) around  $\not{k} = M$ , we find that the divergences appear up to infinite order in powers of  $(\not{k} - M)/M$ . This is the same low-energy expansion employed for the lagrangian [4,14,15] except that we use the equations of motion [3,10,11] to remove higher derivatives on the nucleon fields in favor of multiple  $\pi N$  and  $NN$  interaction terms. For example, counterterms such as  $\frac{1}{M} \overline{N} (\not{\partial} - M)^2 N$  are reduced to other interaction terms by the nucleon equation of motion. Thus, we can absorb all the divergences into the infinite number of parameters in the lagrangian. As emphasized before, this does not invalidate the power counting even though we need an infinite number of counterterms for any loops.



Note further that  $\Sigma_{\pi N}^{\overline{\text{MS}}}$ , the self-energy in the  $\overline{\text{MS}}$  scheme at  $\mu = m_\pi$ , is non-analytic and so it cannot be absorbed into the parameters of the lagrangian. This result is consistent with the expectation that the parameters should contain only high-energy contributions. Finally, taking the limit of infinite nucleon mass, we can verify straightforwardly that Eq. (8) reduces to the corresponding result in heavy-baryon ChPT, as in e.g. Ref. [18].

It is reassuring to verify that the hard part can be absorbed into the parameters of the lagrangian. Performing the integral in Eq. (6) directly, we obtain

$$\begin{aligned} (1 - \hat{S})\Sigma_{\pi N}(k) = & \frac{3g_A^2}{2f_\pi^2}M^2\left(M + \frac{k^2 + M^2}{2k^2}\not{k}\right)\left(L + \frac{1}{32\pi^2}\ln\frac{M^2}{\mu^2}\right) \\ & - \frac{3g_A^2}{64\pi^2f_\pi^2}(\sqrt{k^2} - \omega)\left[\frac{k^2 - M^2}{k^2}\omega\not{k} - \frac{m_\pi^2}{\sqrt{k^2}}(\not{k} + M)\right] \\ & \times \left[32\pi^2L + \ln\frac{M^2}{\mu^2} - 1 + \sum_{l=1}^{\infty}\frac{2}{2l-1}\frac{(\omega^2 - m_\pi^2)^l}{(\sqrt{k^2} - \omega)^{2l}}\right], \end{aligned} \quad (12)$$

which is indeed expandable in powers of  $(\not{k} - M)/M$ . Notice, however, that the first term in Eq. (12) is of  $O(M)$  and the second one is of  $O(Q^2/M)$ , both of which are of higher order than the soft part, which is of  $O(Q^3/M^2)$ . This result is consistent because the hard part comes from large-momentum contributions that spoil the power counting. Note that even in the relativistic treatment of GSS [4] the first term in Eq. (12) with  $\not{k} = M$  was absorbed into the bare nucleon mass.

Although the  $\overline{\text{MS}}$  scheme may be sufficient for the nonrenormalizable higher-order terms, we should perform mass and wave-function renormalizations on the nucleon mass shell because the nucleons are physically observed. Thus, we need further mass and wave-function counterterm subtractions to obtain the renormalized self-energy:

$$\hat{R}\hat{S}\Sigma_{\pi N}(k) = \Sigma_{\pi N}^{\overline{\text{MS}}}(k) - \Sigma_{\pi N}^{\overline{\text{MS}}}(k)|_{\not{k}=M} - \frac{\partial}{\partial\not{k}}\Sigma_{\pi N}^{\overline{\text{MS}}}(k)|_{\not{k}=M}(\not{k} - M). \quad (13)$$

Note that we have adopted the counterterm method of renormalization by starting with physical masses and couplings and then adding counterterms. To unveil the chiral expansion of the nucleon mass we can use

$$M = M_0 + \Sigma_{\pi N}(k)|_{\not{k}=M}, \quad (14)$$

where  $M_0$  denotes the sum of the bare nucleon mass and the bare contributions from the symmetry-breaking contact terms. Using Eqs. (8), (9), and (12) in Eq. (14), we can obtain the same result as that in GSS [4]. In particular, the well-known nonanalytic contribution to the nucleon mass is the leading term in

$$\Sigma_{\pi N}^{\overline{\text{MS}}}(k)\Big|_{\not{=M}} = -\frac{3g_A^2 m_\pi^3}{32\pi f_\pi^2} \left(1 - \frac{m_\pi}{2\pi M} + O(m_\pi^2)\right). \quad (15)$$

Finally, let us briefly consider the  $\pi N$  scattering amplitude up to order  $Q^3$ . According to Eq. (5), we need to calculate tree diagrams constructed from vertices with  $d_i + \frac{1}{2}n_i \leq 4$  and the soft parts of one-loop diagrams constructed from vertices with  $d_i + \frac{1}{2}n_i = 2$ . GSS have calculated the diagrams with pions and nucleons [4], but their results are entangled with large contributions from anti-nucleons, which must be absorbed into the parameters to obtain a useful low-energy expansion. With the inclusion of the  $\Delta$  isobar, the number of diagrams further proliferate. The one-loop calculation in our approach is in progress. Here we simply note that our calculation without including the nonanalytic  $O(Q^3)$  loop contributions has produced a good fit to the phase-shift data for energies up to the  $\Delta$ -resonance region [15], whereas the calculation from heavy-baryon CHPT has not yet been satisfactory [19].

I thank P.J. Ellis, R.J. Furnstahl, S. Jeon, J.I. Kapusta, and B.D. Serot for stimulating discussions and useful comments. This work was supported by the Department of Energy under grant No. DE-FG02-87ER40328.

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